Applied Mathematics:

1. Algebra.
2. Trigonometry.
3. Co-ordinate geometry.
4. Calculus :
5. Integration.
6. Vectors. & Graphs

ALGEBRA

* Quadratic Equation

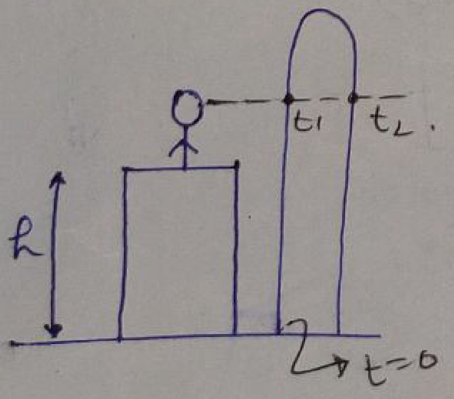
· Equation: An equation is a mathematical statement that two things are equal. It consists of two expressions one of each side of an 'equal' sign.

eg $2x+7 \Rightarrow x = -7/2$ → Here one eqn (relation) one unknown and hence one value comes.

simillarly if we have no. of eqns with unknown variable we can find values corresponding to these equation.

eg. $\left.\begin{array}{l} 2x+y = 3 \\ x+y = 2 \end{array}\right\}$ $x = 1,\ y = 1.$

but, let us a consider an example (situation) in which a boy. is standing at a height 'h' from the ground, if we are asking him what is or. at what time you observe this ball passing through your eyesight.



→ Here we be two values of time. so to deal with these problems we study Quadratic equations.

- A equation is said to be quadratic if it is of form.

$$ax^2+bx+c=0$$

→ a, b, c are contt.
→ $x \to$ unknown.
→ $a \neq 0$
→ max power of x is 2 (otherwise eqn will become polynomial).

soln. $\boxed{x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}}$ → x_1, → x_2.

- $x_1 \cdot x_2 = +\dfrac{c}{a}$ (product of roots).
- $x_1 + x_2 = \dfrac{-b}{a}$ (sum of roots).
- $\Delta = b^2-4ac$
  - → $\Delta \neq 0$ $x_1 \neq x_2$.
  - → $\Delta = 0$ $x_1 = x_2$.
  - → $\Delta = (-ve)$ solution does not exists ∵ in physics.

eg. $2x^2-x-3=0$

Sol $a=2$, $b=-1$, $c=-3$

$\Delta = b^2-4ac \Rightarrow \Delta = (-1)^2 - 4(2)(-3) \Rightarrow \Delta = 25$

∵ $x_1 x_2 = \dfrac{c}{a} \Rightarrow \dfrac{-3}{2}$ (product)

$x_1+x_2 = -b/a = -\dfrac{(-1)}{2} \Rightarrow \dfrac{1}{2}$. sum of roots.

Now checking above results.

$x_1 = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_1 = \dfrac{-(-1)+\sqrt{25}}{2(2)} \Rightarrow \dfrac{6}{4}$

$\boxed{x_1 = \dfrac{3}{2}}$

$x_2 = \dfrac{-(-1)-25}{2(2)}$

$\boxed{x_2 = -1.}$

* Binomial (Theorem) Expression.

if a expression consist of two terms can be considered as binomial expression.

eg. $(a+b)$, $(x+y)^n$, $(2x-3y)^{4/3}$, $(a^2+b^3)^4 \Rightarrow (P+Q)^4$.

Theorem:

$$(a+b)^n = a^n + n a^{n-1} b^1 + \frac{n(n-1)}{2\times 1} a^{n-2} b^2 + ---$$

$$(a+b)^n \cong a^n + n a^{n-1} b \Rightarrow a^n (1 + n a^{-1} b).$$

$$(a+b)^n \cong a^n (1 + n\, b/a) \;;\; \text{if } b/a \to 0.$$

$$\boxed{(1+x)^n \cong 1+nx}^{*} \;;\; \text{if } x \to 0$$ (i.e. x must be very small value)

so using binomial expression. we can simplify. two term having large power into simpler one following example will show:-

eg. Calculate $(1001)^{1/3}$

Sol we will set $(1001)^{1/3}$ on binomial theorem. which is

$$(1+x)^n = 1+nx.$$

so $(1000+1)^{1/3}$.

$$= \left[1000\left(1+\frac{1}{1000}\right)\right]^{1/3}$$

$$= (1000)^{1/3} \left(1+\frac{1}{1000}\right)^{1/3}$$

$$= \underbrace{10\left(1+\frac{1}{1000}\right)^{1/3}}$$

this is of the $(1+x)^n$ form

$$= 10\left(1+\frac{1}{3}\cdot\frac{1}{1000}\right)$$

$$= 10\left(1+(0.3)(0.001)\right)$$

$$= 10\left(1+0.0003\right)$$

$$= 10\,(1.0003)$$

$$= 10.003 \quad \text{Ans}$$

$(1001)^{1/3} \longrightarrow$ Reduced to $\rightarrow (10.003)$

* Logarithm

Formula

① $\log(mn) = \log m + \log n$ ④ $\log_b b^x = x$

② $\log\left(\frac{m}{n}\right) = \log m - \log n$.

③ $\log a^n = n \log a$.

NOTE: By convention logarithms to the base 10 simply written as log instead of $\log_{10}$.

eg. Find value using logarithm.

$y = 8^{27} \rightarrow$ taking log $\Rightarrow \log y = \log 8^{27}$

$\therefore \log y = 27 \log 8$ [using ③]

$\log y = 27 \log 2^3$

$\log y = 27 \times 3 \log 2$

we will see value of log 2 from log table given in book. which is $\log 2 = 0.3010$

$\log y = 27 \times 3 \times 0.3010$

$\log y = 24.381$

Now we will take Antilog value (from antilog table) to get the final value of y

$\log y = 24.381 \rightarrow$ Antilog $\Rightarrow y = 1.387$

Note: $\log_b 1 = 0$

eg based on $\log(mn) = \log m + \log n$.

find $\log 20 = \log_{10} 20 \Rightarrow$ here we have to find what should be powes of 10 so that value is 20.

$= \log_{10} 2 \times 10$

$= \log_{10} 2 + \log_{10} 10$

$= \log_{10} 2 + 1.$

$= 0.3010 + 1$

$\log_{10} 20 = 1.3010$ Ans.

Simillarly: eg.1 $\log_2 4 = 2$

$2^n = 4 \Rightarrow n = 2.$

eg 2 $\log_3 9 = 2$

$3^n = 9$

$n = 3$

• Natural log: A natural logarithm is a logarithm with base e where [e = irrational number, $\to$ Value $\simeq 2.71$]

This number has important application in calculus.

• Notation of Natural logarithm $\Rightarrow \log_e x$ (or sometimes) $\ln x$.

• examples on Natural log

find $\log_e 10 \Rightarrow 2.303 \log_{10} 10.$

$\Rightarrow 2.303 \times 1 \Rightarrow 2.303$ Ans

• Change of base Formula

Calculator can calculate logarithms to the base 10 or e, so change of base formula allows us to calculate logarithm of any base with our calculator.

$$\boxed{\log_a b = \frac{\log_c b}{\log_c a}.}$$

eg. if $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$ then find.

$\log_{10} \sqrt{18} = ?$

Solution: $\because \log_{10} \sqrt{18}$

$$= \log_{10} 18^{1/2} \Rightarrow \frac{1}{2} \log_{10} 18 \Rightarrow \frac{1}{2} \log_{10} (3^2 \times 2)$$

$$= \frac{1}{2}\left[\log_{10} 3^2 + \log_{10} 2\right] \Rightarrow \frac{1}{2}\left[2 \log_{10} 3 + \log 2\right].$$

$$\Rightarrow \frac{1}{2}\left[2(0.4771) + (0.3010)\right] \Rightarrow 0.6276$$

Examples on change of base

① $\log_{10} 42 = \dfrac{\log_5 42}{\log_5 42}$ (if we know value in base 5).

② $\log_3 40 = \dfrac{\log_{10} 40}{\log_{10} 3}$ (if we know value in base 10).

* Componendo & Dividendo (Rule).

$$\frac{P}{Q} = \frac{a}{b} \Rightarrow \frac{P+Q}{P-Q} = \frac{a+b}{a-b}.$$

eg. $\frac{1}{2} = \frac{3}{6} \Rightarrow \frac{1+2}{1-2} = \frac{3+6}{3-6} \Rightarrow \frac{9}{-3} \Rightarrow -3$ Ans

* Series.

(i) Arithmetic Progression (AP)

→ n^{th} term ⇒ $\boxed{a+(n-1)d = a_n}$

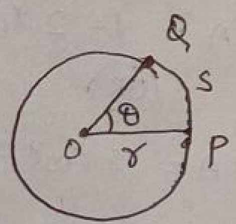
→ sum ⇒ $\boxed{S_n = \frac{n}{2}\left[2a+(n-1)d\right]}$

(ii) Geometric Progression (G.P)

→ n^{th} term ⇒ $\boxed{ar^{n-1}}$

→ sum → $\boxed{S_n = \frac{a(1-r^n)}{1-r}}$

* Trigonometry.



- Consider a particle moves from $P \rightarrow Q$. along a circle of radius r Then,

$$\text{Angle } (\theta) = \frac{\text{Arc length (PQ)}}{\text{Radius} \quad (r)} = \frac{s}{r} \Rightarrow \boxed{s = r\theta}$$

if Arc length = Radius of circle Then $\theta = \frac{\cancel{r}}{\cancel{r}} \Rightarrow \boxed{\theta = 1 \text{ radian}}$

- Radian is the unit of measuring angle.
- When a body completes one revolution

$\theta = 2\pi$ rad.

2π rad $= 360°$

2×3.14 rad $= 360°$

$1 \text{ rad} = \frac{360°}{2 \times 3.14}$

$\boxed{1 \text{ rad} = 57.3°}$

OR

OR

$360° = 2\pi$ rad.

$1° = \frac{\cancel{2}\pi}{\cancel{360}}$ rad

$\boxed{1° = \frac{\pi}{180°} \text{ rad.}}$

We will use these relation for rad ⇌ degree

* Trigonometrical Identities

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
3. $1 + \tan^2\theta = \sec^2\theta$.
4. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
5. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
6. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
7. $\cos(A-B) = \cos A \cos B + \sin A \sin B$
8. $\sin 2A = 2 \sin A \cos B$

$$\left.\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &\text{OR} \\ \cos 2A &= 1 - 2\sin^2 A \\ &\text{OR} \\ \cos 2A &= 2\cos^2 A - 1 \end{aligned}\right\} \text{Imp}$$

* Commonly used Values.

∠s →	0°	30°	45°	60°	90°
sin θ	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1.
cos θ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
tan θ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

Simillarly we can find for other ratios as :-

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot = \frac{1}{\tan\theta}.$$

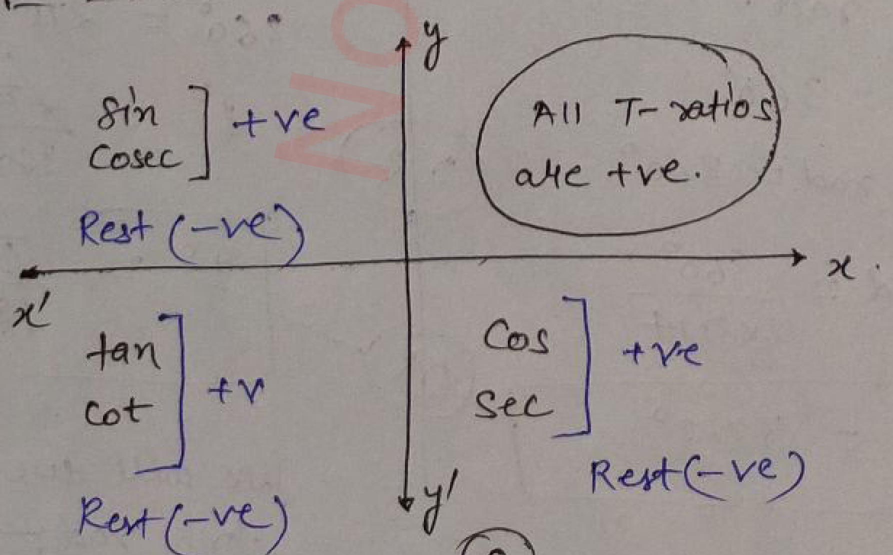
* Some commonly used values (Remember it).

$\sin 37° = \frac{3}{5}$ $\sin 53° = \frac{4}{5}$

$\cos 37° = \frac{4}{5}$ $\cos 53° = \frac{3}{5}$

$\tan 37° = \frac{3}{5}$ $\tan 53° = \frac{4}{3}$

* Sign of Trigonometrical Ratio in any quadrant



* **Function:** Function is a rule of relationship between two variables in which one is assumed to be a dependent variable and other independent variable.

eg. The temprature at which water boils depends upon the elevation above the sea level, here the elevation is independent but the temprature depends upon elevation.

→ So function is a mathematical form which describes this relation between elevation (height) h with temprature (t).

→ And How this variation changes can be studied by Calculus

→ Calculus ⇒ (Differentiation + Integration) etc combinedely called as calculus.

* **Differentiation :—**

The purpose of differential calculus (differentiation) is to study the nature (i.e. increase or decrease).

(a) Quantity → ~~Anything~~ Anything that can be measured called as quantity.

1(c) Properties of Derivatives

- $\frac{d}{dx} k\, f(x) = k \frac{d f(x)}{dx}$.

- $\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d f(x)}{dx} + \frac{d\, g(x)}{dx}$

- $\frac{d}{dx}\left[a f(x) + b\, g(x)\right]$

$= a \cdot \frac{d}{dx} f(x) + b \cdot \frac{d}{dx} g(x)$

· Product Rule.

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

· Quotient Rule.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$

· Derivatives of some important functions.

(a) $\frac{d}{dx}(\sin x) = \cos x$

(b) $\frac{d}{dx}(\cos x) = -\sin x$

(c) $\frac{d}{dx}(\tan x) = \sec^2 x$.

(d) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(e) $\frac{d}{dx}(\sec x) = \tan x \sec x$

(f) $\frac{d}{dx}(\operatorname{cosec} x) = -\cot x \operatorname{cosec} x$.

(g) $\frac{d}{dx} x^n = nx^{-1}$

(h) $\frac{d}{dx}(\log_a x) = \frac{1}{x}\log_a e$.

(i) $\frac{d}{dx}(\log_e x) = \frac{1}{x}\log_e e = \frac{1}{x}$.

(j) $\frac{d}{dx}(e^x) = e^x$.

(k) $\frac{d}{dx}(\text{constant}) = 0$.

eg. if $y = x^5$ find $\frac{dy}{dx}$.

Sol using $\frac{d}{dx} x^n = n x^{n-1}$ we have.

$$\frac{dy}{dx} = \frac{d}{dx} x^5 = 5x^{5-1}$$

$$= 5x^4 \quad \underline{\underline{\text{Ans}}}$$

eg. if $y = 3x^2 + 2x$ find $\frac{dy}{dx}$.

Sol $\frac{dy}{dx} = \frac{d}{dx}(3x^2 + 2x)$

$$\frac{dy}{dx} = \frac{d}{dx} 3x^2 + \frac{d}{dx} 2x$$

$$= 3 \cdot \frac{d}{dx} x^2 + 2 \cdot \frac{d}{dx} x$$

$$= 3 \cdot 2x + 2.$$

$$= 6x + 2 \quad \underline{\underline{\text{Ans}}}$$

eg. Find the derivatives $\left(\frac{dy}{dx}\right)$ of $y = (x^2+1)(x^3+3)$

Sol (using product Rule)

$$\frac{d}{dx}(x^2+1)(x^3+3)$$

$$= (x^2+1)\frac{d}{dx}(x^3+3) + (x^3+3)\frac{d}{dx}(x^2+1)$$

$$= (x^2+1)\left[\frac{d}{dx}x^3 + \frac{d}{dx}(3)\right] + (x^3+3) \cdot \left[\frac{d}{dx}x^2 + \frac{d}{dx}(1)\right]$$

$$= (x^2+1)\left[(3x^2) + 0\right] + (x^3+3)(2x)$$

$$= (x^2+1)(3x^2) + (x^3+3)(2x).$$

$$= (3x^4 + 3x^2) + 2x^4 + 6x.$$

$$= 5x^4 + 3x^2 + 6x.$$

eg: if $y = \frac{\sin x}{x+\cos x}$, find $\frac{dy}{dx}$. → based on Quotient Rule.

Sol

$$\frac{dy}{dx} = \frac{(x+\cos x)\cdot\frac{d}{dx}(\sin x) - \sin x \frac{d}{dx}(x+\cos x)}{(x+\cos x)^2}$$

$$\frac{dy}{dx} = \frac{(x+\cos x)(\cos x) - \sin x\left[\frac{d}{dx}x + \frac{d}{dx}\cos x\right]}{(x+\cos x)^2}$$

$$= \frac{(x+\cos x)(\cos x) - \sin x(1-\sin x)}{(x+\cos x)^2}$$

$$= \frac{x\cos x + \cos^2 x - \sin x + \sin^2 x}{(x+\cos x)^2}$$

$$= \frac{x\cos x - \sin x + \underbrace{\sin^2 x + \cos^2 x}_{(=1)}}{(x+\cos x)^2}$$

$$= \frac{x\cos x - \sin x + 1}{(x+\cos x)^2}$$ Ans.

* Integration -

→ Integration is inverse operation of differentiation.

- Standard formulae for integration.

① $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ (Note: C = Constant)

② $\int (f(x)+g(x))\,dx = \left[\int f(x)\,dx + \int g(x)\,dx\right] + c$.

③ $\int c\,dx = cx + c$

④ $\int x^{-1}\,dx = \log_e x + C$.

⑤ $\int e^x\,dx = e^x + C$.

⑥ $\int \sin x\,dx = -\cos x + C$.

⑦ $\int \cos x\,dx = \sin x + C$.

⑧ $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$.

⑨ $\int \sec^2 x\,dx = \tan x + C$.

Q= Integrate the following w.r.t. x

(i) x^3 (ii) $x - \frac{1}{x}$.

Sol (i) $y = x^3$

$= \int y\,dx$

$= \int x^3\,dx$

$= \frac{x^{3+1}}{3+1}$

$= \frac{x^4}{4} + c$. Ans

(ii) $y = x - \frac{1}{x}$.

$\int y\,dx = \int \left(x - \frac{1}{x}\right) dx$

$= \int x\,dx - \int \frac{1}{x}\,dx$

$= \frac{x^{1+1}}{1+1} - \ln x + c$.

It is (Natural logarithm)

* Definite Integral

- **Algebraic method to Evaluate Definite integral.**

$$\int_a^b f(x)\,dx = \Big[I(x)\Big]_a^b = I(b) - I(a)$$

Integrated form of function $f(x)$.

Where, $a, b \Rightarrow$ limits.

eg. Evaluate $y = x^2$ for limit 3, 4

Sol

$$\int y\,dx = \int_3^4 x^2\,dx.$$

$$= \left[\frac{x^{2+1}}{2+1}\right]_3^4$$

$$= \left[\frac{x^3}{3}\right]_3^4$$

$$= \frac{1}{3}\left[x^3\right]_3^4$$

$$= \frac{1}{3}\left[(4)^3 - (3)^3\right]$$

$$= \frac{1}{3}(64 - 27)$$

$$= \frac{1}{3}(37)$$

$$= 12.33 \quad \text{Ans}$$

eg. Evaluate. $I = \int_{-\pi/2}^{\pi/2} \cos x\,dx.$

Sol

$$= \int_{-\pi/2}^{\pi/2} \cos x\,dx$$

$$= \Big[\sin x\Big]_{-\pi/2}^{\pi/2}$$

$$= \sin\left(\pi/2\right) - \sin\left(-\pi/2\right)$$

$$= 1 - \left(-\sin\left(\pi/2\right)\right)$$

$$= 1 + \sin \pi/2$$

$$= 1 + 1$$

$$= 2 \quad \text{Ans}$$

because.

$$= \sin(-\theta)$$
$$= -\sin\theta$$

Chapter-2 : Units and Measurement

* **Physical Quantity:** A quantity which can be measured and by which various physical happening can be explained and expressed in form of laws is called a physical quantity. eg mass, time etc

$$\text{Physical Quantity } (Q) = \left(\text{magnitude or Numerical Value}\right) \times \text{unit}$$

or,

$$\boxed{Q = nU}$$

where, n = magnitude or numerical value

U = unit.

if we express a given physical Quantity in another unit then unit ~~ca~~ changes hence magnitude also changes

means $\boxed{nU = \text{constant}}$ or $\boxed{n_1U_1 = n_2U_2 = \text{constant}}$

meaning → $\underbrace{1}_{n_1}\ \underbrace{kg}_{U_1} = \underbrace{1}_{n_2} \times \underbrace{1000\,gm}_{U_2}$.

→ Unit is different.

but these unit contain values of each other conversion.

*. **Fundamental and derived Quantities**

(a) Fundamental Quantities ⇒ they are exist by natural i.e. they are absolute they do not require other Quantities to express eg. length, mass, time, current, luminous intensity, temprature etc.

(b) Derived Quantities ⇒ Quantities which are derived from fundamental Quantities by suitable mathematical methods eg m/s for acceleration, force (newton). etc.

Unit: The word unit as used in ~~phys~~ Physics refers to the standard measure of quantity.

System of units

- CGS: Centimetre, gram, second
- MKS: metre, Kilogram, second
- FPS: foot, Pound, second.

* **S.I system** ⇒ It is known as international system of units. It is applied in whole physics

Quantity	Name of unit	Symbol	Dimension
1. Length	metre	m	→ L
2. Mass	Kilogram	kg.	→ M
3. Time	second	S	→ T
4. Electric current.	ampere	A	→ A
5. Temprature	Kelvin	K	→ Θ
6. Amount of substance.	~~mode~~ mole.	mol	→ mol.
7. Luminous Intensity.	Candela.	Cd.	→ Cd

Note: Supplementary units (i) Radian (rad) plane angle.
(ii) Steradian (sr) solid angle.

Practical units: i) light year ii) horse power (hp)
iii) mile (1 mile = 1.6 km). etc.

* Dimensions of a Physical Quantity.

→ Expressing Quantities in terms of base quantities.

eg. Force = mass x acceleration ⇒ $F = ma$.

$$\text{Force} = kg \cdot \frac{m}{s^2}$$

$$\text{Force} = M \frac{L}{T^2} \Rightarrow \boxed{\text{Force} = MLT^{-2}}$$

eg. find the dimensional formula (i) $F = G\frac{m_1 m_2}{r^2}$ for G (ii) Pressure.

(i) $F = G\frac{m_1 m_2}{r^2}$

for G: $G = \frac{Fr^2}{m_1 m_2}$

$$G = \frac{\cancel{M}LT^{-2}L^2}{\cancel{M}M}$$

$$\boxed{G = M^{-1}L^{3}T^{-2}.}$$

(ii) Pressure = $\frac{\text{Force}}{\text{Area}}$.

$$= \frac{MLT^{-2}}{L^2}$$

$$\boxed{\text{Pressure} = ML^{-1}T^{-2}}$$

* Principle of homogeneity:

It states that a quantity can be added or substracted to the same i.e. length can be added length only eg $L+L=L$ or $L-L=L$ Similarly $M-M=M$ or $M+M=L$.

* Dimensional correctness of a given physical relation

In this we check dimension of a given physical Quantity i.e. its correctness.

Q- Check the correctness of given physical Quantity.

(i) $F = \frac{mv^2}{r^2}$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $T = 2\pi\sqrt{\frac{l}{g}}$.

(i) $F = \frac{mv^2}{r^2}$

LHS $\Rightarrow$ $F = MLT^{-2}$

RHS: $\frac{mv^2}{r^2} = \frac{M[LT^{-1}]^2}{L^2} = \frac{M\cancel{L^2}T^{-2}}{\cancel{L^2}} \Rightarrow MT^{-2}$.

LHS $\neq$ RHS $\rightarrow$ hence given expression is dimensionally wrong.

(ii) $s = ut + \frac{1}{2}at^2$

LHS: $s = L$

RHS: $ut + \frac{1}{2}at^2 = [LT^{-1}][T] + LT^{-2}T^2$

$= [L\cancel{T^{-1}}\cancel{T}] + [L\cancel{T^{-2}}\cancel{T^2}]$

$= L + L = L = $ LHS.

$\therefore$ LHS = RHS $\rightarrow$ hence given expression is dimensionally correct.

(iii) $T = 2\pi\sqrt{\frac{l}{g}}$.

LHS: T (time) $= [T]$

RHS: $2\pi\sqrt{\frac{l}{g}} = \sqrt{\left[\frac{\cancel{L}}{\cancel{L}T^{-2}}\right]} \Rightarrow \sqrt{T^2} = T$.

LHS = RHS $\rightarrow$ Correct ✓

* Conversion of Physical Quantity from one system to another

$$\because \quad n_1 [u_1] = n_2 [u_2].$$

$$n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$$

or, $$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c.$$

eg. Convert 1N into dyne.

Sol $1N = [MLT^{-2}] \longrightarrow$ gives $a=1,\ b=1,\ c=-2$. or $1N = 1\,kg\ m/second^2$.

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c.$$

$\because$ dyne is CGS unit $\to$ cm, gm, s.

$$n_2 = 1 \left[\frac{kg}{gm}\right]^1 \left[\frac{m}{cm}\right]^1 \left[\frac{s}{s}\right]^{-2}$$

$$\cancel{1\ \frac{kg}{100}} \quad n_2 = 1 \left[\frac{1000\,\cancel{gm}}{\cancel{gm}}\right]^1 \left[\frac{100\,\cancel{cm}}{\cancel{cm}}\right]^1 \left[\frac{\cancel{s}}{\cancel{s}}\right]^{-2}$$

$$n_2 = 1 \times (1000)(100)$$

$$\therefore 1N = 10^5 \text{ dyne}$$

Q= Convert 1 MW power in new system in which mass 10kg, 1 dm. and 1 minute are used as basic units.

Sol Power $= ML^2T^{-3} \longrightarrow$ this gives, $a=1,\ b=2,\ c=-3.$

$$\because \quad n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c.$$

$$n_2 = 1 \times 10^6 \left[\frac{1\cancel{kg}}{10\cancel{kg}}\right]^1 \left[\frac{1m}{1dm}\right]^2 \left[\frac{1s}{1min}\right]^{-3}$$

$$= 1 \times 10^6 \left[\frac{1}{10}\right]^1 \left[\frac{10\cancel{dm}}{1\cancel{dm}}\right]^2 \left[\frac{1\cancel{s}}{60\cancel{s}}\right]^{-3}$$

$$= 1 \times 10^6 \left(\frac{1}{10}\right) (10)^2 \left(\frac{1}{60}\right)^{-3}$$

$$= 1 \times 10^6 \left(\frac{1}{10}\right) (10)^2 (60^3)$$

$$= 2.16 \times 10^{12} \text{ unit}$$

* **Deriving new relation using dimensions.**

Q: Consider a simple pendulum having a bob attached to a string that oscillates under the action of force of gravity. Suppose that the period of oscillation of the simple pendulum depend on its length (l) mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using method of dimension.

Sol

$$T \propto l^x$$

$$T \propto g^y$$

$$T \propto m^z$$

$$T \propto l^x g^y m^z$$

$$T = k\, l^x g^y m^z \quad \text{—— (A)}$$

6

$$[M^0 L^0 T^1] = [L^1]^x [L^1 T^{-2}]^y [M^1]^z$$

$$M^0 L^0 T^1 = L^x L^y T^{-2y} M^z$$

$$M^0 L^0 T^1 = L^{x+y} T^{-2y} M^z$$

equating we have $x+y=0, \quad -2y = 1, z=0$

$\therefore -2y = 1 \Rightarrow \boxed{y = -\frac{1}{2}}$

$\because x+y=0 \Rightarrow x - \frac{1}{2} = 0 \quad \boxed{x = \frac{1}{2}}$ and $\boxed{z=0}$

putting value in (1)

$$T = k\, l^{1/2} g^{-1/2} \Rightarrow \boxed{T = k\sqrt{\frac{l}{g}}}$$

*. Errors and measurements:

Error: It is difference between measured value and true value.

(1) Absolute error → It is the difference between avg of measured value and individual measure values.

i.e. Let a quantity is measured a_1, a_2, a_3, a_4 --- --- a_n times then

arithmetic mean $a_m = \dfrac{a_1 + a_2 + a_3 + \cdots a_n}{n}$

(takes 'As' True value)

Now, we have a_m which is mean of many measured value, so we consider it true value.

Now, definition of absolute error says.

$$\text{absolute error } (\Delta a) = \underbrace{a_m}_{\text{mean value or true value}} - \underbrace{a_1}_{\text{individually measured value of physical quantity}}$$

$\therefore$ for each measurement errors will be

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\boxed{\Delta a_n = a_m - a_n}$$

This absolute error may be +ve / -ve.

(2) Mean absolute error ⇒ It is the mean of all the calculate absolute error i.e. Δa_1, Δa_2, Δa_3 ---- Δa_n.

i.e.
$$\boxed{\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + --- |\Delta a_n|}{n}}$$

in calculating mean of absolute error we have taken +ve values i.e. modulus because it can be +ve or -ve

hence final measurement of physical Quantity can be expressed as

$$a = (a_m \pm \overline{\Delta a})$$

$(a_m + \overline{\Delta a})$ and $(a_m - \overline{\Delta a})$

③ <u>Relative error or fractional error</u>

$$\text{Relative error} = \frac{\text{mean absolute error}}{\text{mean Value}} = \frac{\overline{\Delta a}}{a_m}.$$

④ <u>Percentage error</u> $= \frac{\overline{\Delta a}}{a_m} \times 100$

* <u>Propagation of error</u> –

(a) Error in sum $x = a + b$ $\qquad x = \frac{\Delta a + \Delta b}{a+b} \times 100\ \%$

(b) Error in difference. $x = a - b$ $\qquad x = \frac{\Delta a + \Delta b}{a-b} \times 100\ \%$

(c) Error in product. $x = a \times b$ $\qquad \frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

(d) Error in Division. $x = a/b$ $\qquad \frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

(e) Error in quantity raised to some power $x = \frac{a^n}{b^m}$

$$\boxed{\frac{\Delta x}{x} = \pm\left[n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right]}$$

Q=2.13
NCERT
Page ⇒ 36

$\because \quad P = \frac{a^3 b^2}{(\sqrt{c}\, d)}$

$$\therefore \frac{\Delta P}{P} = 3\frac{\Delta a}{a} + 2\cdot\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}$$

$$\underbrace{\left(\frac{\Delta P}{P}\times 100\right)\%}_{\text{percentage error}} = \left(3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}\right)\times 100\%$$

$= 3\times 1 + 2\times 3 + \frac{1}{2}\times 4 + 2$

$= 13\%$ Ans

after Rounding off $3.763 \rightarrow 3.8$ Ans